Application of PDE control to problems in drilling

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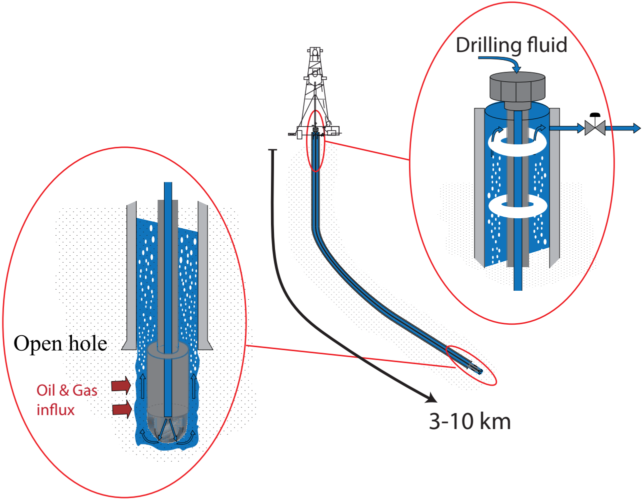
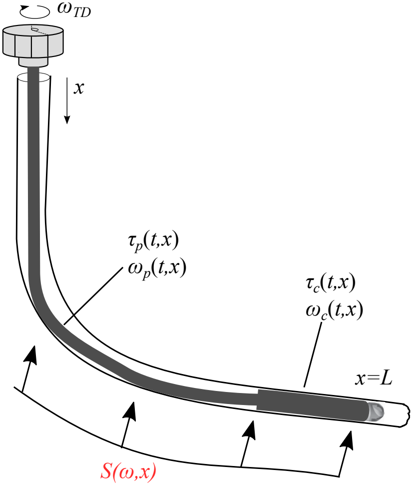
Research goals

- Develop application oriented results for control and estimation of PDEs
  - For systems with important distributed effects
  - Taking into account delays / wave propagation
- Apply results to problems in drilling

Application domain: drilling



Torsional Model: ODE-PDE-PDE



- Topside BC

$$\dot{\omega}_{TD} = \frac{1}{I_{TD}}(\tau_m - \tau(0,t))$$

- Distributed wave eq.: $i \in \{p, c\}$

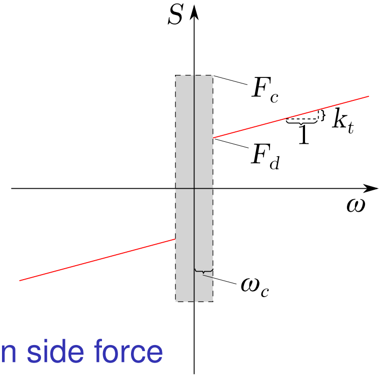
$$\frac{\partial \tau_i(t,x)}{\partial t} + J_i G \frac{\partial \omega_i(t,x)}{\partial x} = 0$$

$$J_i \rho \frac{\partial \omega_i(t,x)}{\partial t} + \frac{\partial \tau_i(t,x)}{\partial x} = -S(\omega_i, x),$$

- Coupling

$$\omega_p(L_p, t) = \omega_c(0, t)$$
$$\tau_p(L_p, t) = \tau_c(0, t)$$

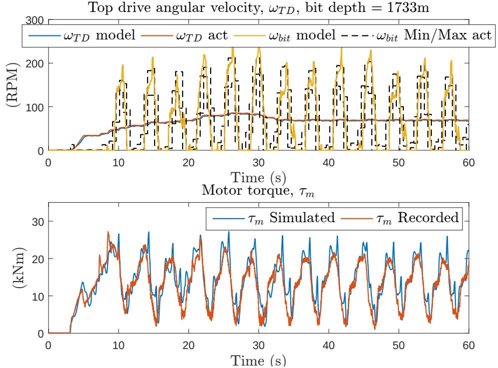
Model: Side force

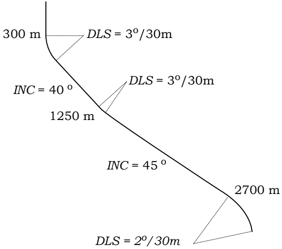


Coulomb friction side force

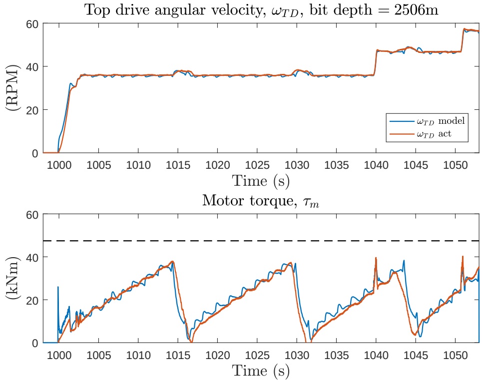
$$\begin{cases} S(\omega, x) = F_d(x), & \omega \geq \omega_c \\ S(\omega, x) \in [-F_c(x), F_c(x)] & |\omega| < \omega_c \\ S(\omega, x) = F_d(x), & \omega \leq -\omega_c \end{cases}$$

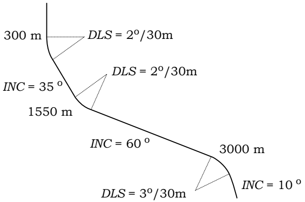
Field data ex 1. 1,733 meter [Aarsnes, UJF and Shor, RJ 2018]





Field data ex 2: 2,2506 meter [Aarsnes, UJF and Shor, RJ 2018]





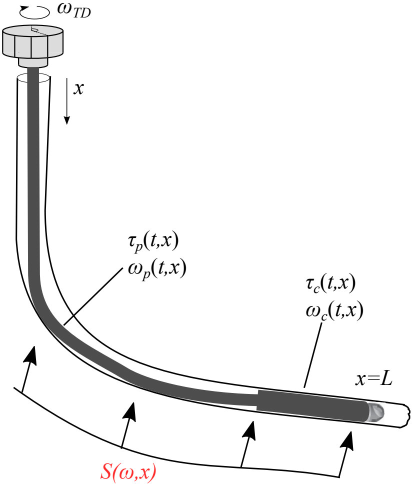
Control and estimation objectives

1. Control: Avoid “stick-slip” oscillations

2. Estimate: Downhole state and side forces

3. Will use approximate *ODE-PDE-ODE* model

Approximate Model: ODE-PDE-ODE



- Topside BC

$$\dot{\omega}_{TD} = \frac{1}{I_{TD}}(\tau_m - \tau(0,t))$$

- Drillstring

$$\frac{\partial \tau_p(t,x)}{\partial t} + J_p G \frac{\partial \omega_i(t,x)}{\partial x} = 0$$

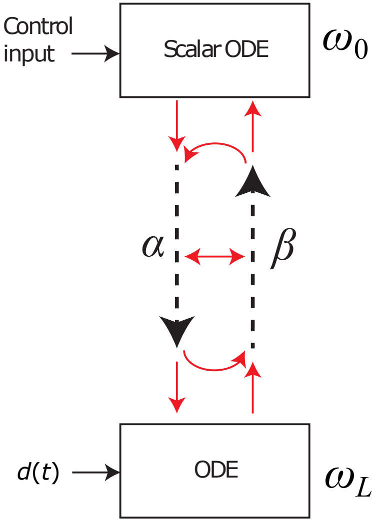
$$J_p \rho \frac{\partial \omega_p(t,x)}{\partial t} + \frac{\partial \tau_p(t,x)}{\partial x} = 0,$$

- Lumped collar

$$\dot{\omega}_L = \frac{1}{I_{\text{BHA}}}\Big(\tau_p(L_p,t) - d(t)\Big),$$

$$d(t) \approx \int_0^L S(\omega,x)$$

Model structure



Riemann invariants

$$\alpha = \omega + \frac{c_t}{JG}\tau, \quad \beta = \omega - \frac{c_t}{JG}\tau,$$

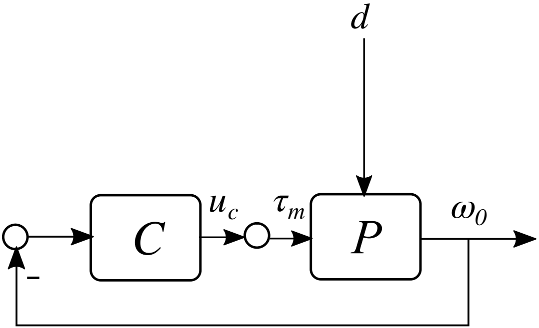
Parametrize measured state, ω_0, and control, τ_m, using desired output $z(t) = \omega_L(t)$.

$$\omega_0 = \frac{z(t+t_D) + z(t-t_D)}{2} + \frac{\dot{z}(t+t_D) - \dot{z}(t-t_D)}{2a_L}, \tag{1}$$

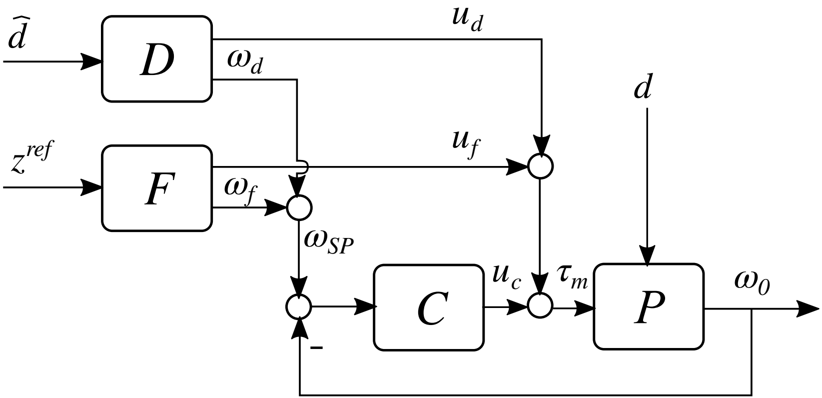
$$\frac{2}{I_{TD}}\tau_m = \dot{\omega}_0 + a_0\omega_0 + \frac{a_0}{a_L}\dot{z}(t-t_D) - a_0 z(t-t_D). \tag{2}$$

Trajectory planning for $z(t)$ is easy....

Disturbance cancellation + trajectory planning

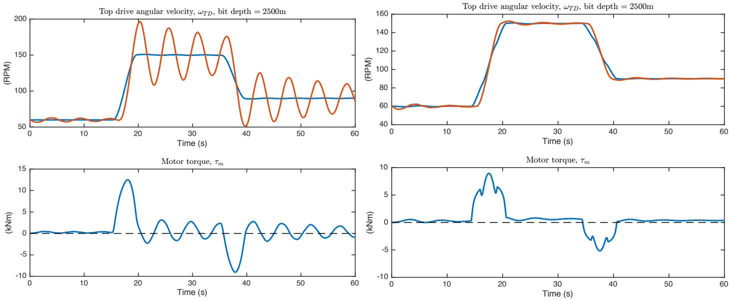


Disturbance cancellation + trajectory planning

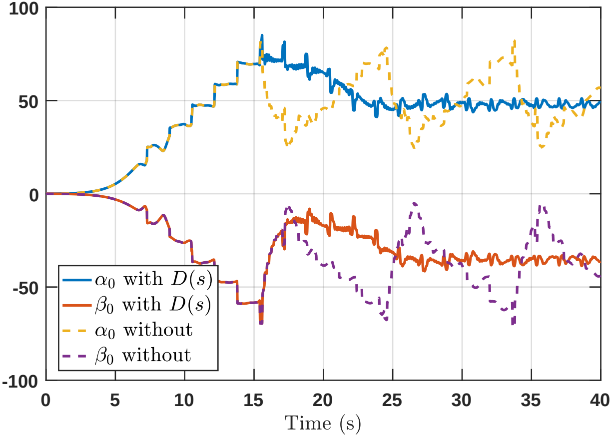


Setpoint change [Aarsnes et al. 2018]

Avoid exciting vibration modes



Feedforward control at startup: Canceling $d(t)$ term



Feedforward control at startup

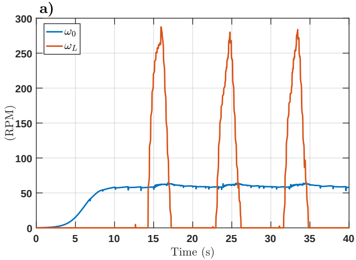


Figure: Nominal case (**left**) with feedforward disturbance cancellation (**right**).

Boundary observer design [Auriol et al. 2018]

- Backstepping transformation, following [Bou Saba et al., 2017]:

$$
\begin{aligned}
u(x,t) &\equiv \alpha(x,t) - \int_x^L K^{uu}(x,y)\alpha(y,t)dy - \int_x^L K^{uv}(x,y)\beta(y,t)dy - G_u(x)^\top \omega_L(t) \\
v(x,t) &\equiv \beta(x,t) - \int_x^L K^{vu}(x,y)\alpha(y,t)dy - \int_x^L K^{vv}(x,y)\beta(y,t)dy - G_v(x)^\top \omega_L(t),
\end{aligned}
$$

with well posed kernels $K^{uu}, K^{uv}, K^{vu}, K^{vv}$ and G_u, G_v to place poles of the distal ODE.

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with well posed kernels K^{uu}, K^{uv}, K^{vu}, K^{vv} and G_u, G_v to place poles of the distal ODE.

- Slightly modified variable change for ω_0, c.f. [Bou Saba et al., 2017] no reflection cancellation term:

$$\sigma_0(t) \equiv \omega_0(t) - c_0^{-1} \int_0^L \left[\tilde{K}_\alpha(y)\alpha(y) + \tilde{K}_\beta(y)\beta(y)\right] dy - c_0^{-1} \tilde{G}^\top \omega_L(t)$$

with $\tilde{K}_\alpha$, $\tilde{K}_\beta$, $\tilde{G}$ functions of the kernels.

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  with $\tilde{K}_\alpha, \tilde{K}_\beta, \tilde{G}$ functions of the kernels.
- Chose control law such that

$$\dot{\sigma}_0(t) = \check{p}\sigma_0(t) + d_0 \frac{\check{p}}{c_0} \beta(0,t)$$
$$u(0,t) = c_0\sigma_0(t) + d_0 v(0,t)$$

  with tuning variable $\check{p} > 0$.

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$$\dot{\sigma}_0(t) = \check{p}\sigma_0(t) + d_0 \frac{\check{p}}{c_0}\beta(0,t)$$
$$u(0,t) = c_0\sigma_0(t) + d_0 v(0,t)$$

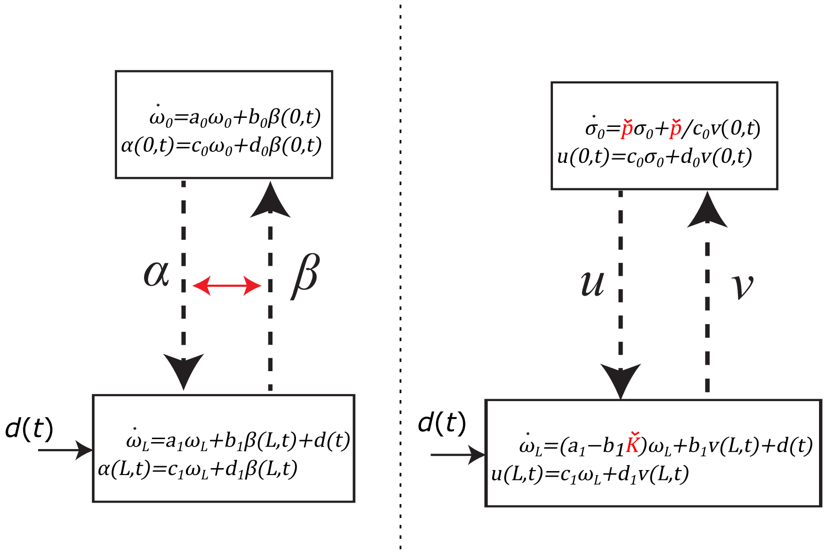
  with tuning variable $\check{p} > 0$.
- Yields "low-pass filtered" reflection (in Laplace domain):

$$u(s,0) = d_0 \frac{s}{s-\check{p}} v(s,0). \tag{3}$$

Boundary observer design [Auriol et al. 2018]

Original system

Target system



Results

Simplified case: $S(\omega, x)$ linear, constant disturbance d

$$\dot{\hat{d}}(t) = \gamma(\hat{\omega}_{TD} - y(t)) \tag{4}$$

- theorem guarantees convergence and robustness to delay mismatch
- degrees of freedom to calibrate the observer
- convergence speed vs. noise sensitivity

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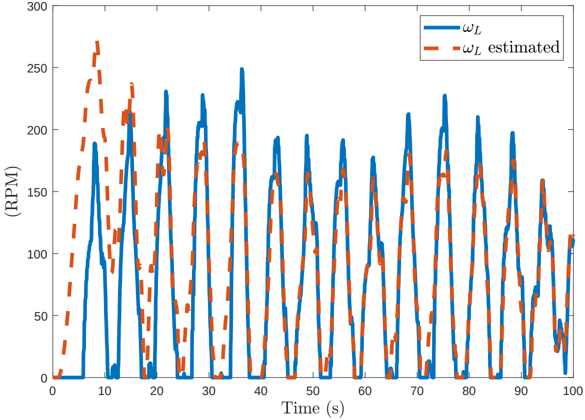
- theorem guarantees convergence and robustness to delay mismatch
- degrees of freedom to calibrate the observer
- convergence speed vs. noise sensitivity

HACK: Friction estimation (*work in progress*)

$$\begin{cases} \hat{d}(t) = \hat{d}_d & \text{if } |\hat{\omega}_L| > \omega_c \\ \hat{d}(t) \in \pm\hat{d}_s & \text{if } |\hat{\omega}_L| < \omega_c \end{cases}$$

Use gains from ‘ODE-PDE-ODE’ backstepping result on ‘ODE-PDE-PDE’ model. Estimate disturbance $d(t)$ to update Cooulomb friction model.



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